CHAIRE D'EXCELLENCE

FONDATION SCIENCES MATHÉMATIQUES DE PARIS

PSL RESEARCH UNIVERSITY PARIS

COURSE DESCRIPTION
SYSTEMS WITH COULOMB-TYPE INTERACTIONS: MEAN-FIELD DYNAMICS AND STATISTICAL MECHANICS

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The goal of this course, targeted at the doctoral level, is to present recent progress in the analysis of statistical mechanics and dynamics of classical Coulomb systems. In order to arrive at those recent developments we will start by reviewing some classical notions and results, which can be instructive for students and young researchers. We will focus on the Coulomb case for simplicity, and explain how some of the results extend to more general singular (inverse power-like) interactions.

The course will be in two parts: one part on the dynamics and one on the statistical mechanics, and we will emphasize several notions that are common to both parts.

We hope the course will be of interest to both analysts and probabilists, as well as researchers interested in mathematical physics in general and in applied mathematics.

The course can be given either in French or in English according to the audience. It has been designed for 8 sessions of 2.5 hours each. Here is a rough schedule.

- Lecture 1: Motivations for Coulomb systems. Main definitions. The notion of Gamma-convergence and equilibrium measure. Connection with the classical obstacle problem. Convergence to the equilibrium measure for minimizers. Splitting the energy to next order.
- Lecture 2: The next order or modulated energy and its electric formulation. Properties of the modulated energy (truncation, monotonicity, coerciveness). The main functional inequality.
- Lecture 3: The dynamical problems. Context and prior results. Notion of mean-field convergence and propagation of chaos. The modulated energy method.
- Lecture 4: The case with noise and global-in-time estimates. Study of fluctuations around the mean-field dynamics.
- Lecture 5: The notion of Large Deviations Principle (LDP). Sanov's theorem. LDP for the empirical measure of the Coulomb gas. First expansion of the free energy and first concentration bounds.
- Lecture 6: The Coulomb renormalized energy. The screening procedure and almost additivity of the energy.
- Lecture 7: Local laws by bootstrap on scales and LDP for empirical fields.
- Lecture 8: The transport method and proof of the Central Limit Theorem for fluctuations.