Comment voter par Internet en toute sécurité ... ou pas ?

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LORIA & Inria Nancy - Grand'Est

Maths en mouvement:
le vote à la loupe
11 juin 2022 – Paris
The County Election, 1852 by George Caleb Bingham
The need for privacy

*Premières expériences d’isoloir (appelé cabine d’isolement) aux élections municipales, en Bretagne. Caricature de Damblans, dans Le Pélérin, 23 novembre 1913.*
Political legally binding Internet elections in Europe:

- parliamentary elections in Switzerland (several cantons)
- parliamentary election in Estonia (all eligible voters)
- municipal and county elections in Norway (selected municipalities, selected voter groups)
- parliamentary elections in France (“expats”)

But also banned in Germany, Ireland, UK

Even more professional elections
E-voting

Essential **security properties** of (e-)voting:

- **Integrity** of the election
- **Secrecy** of the vote
E-voting

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- **Integrity** of the election
- **Secrecy** of the vote

**Warning:**

With Internet voting (like any remote voting) there is no private voting booth!

**Cryptographic protocols to the rescue?**
The Helios e-voting protocol

Verifiable online elections via the Internet

http://heliosvoting.org/

Already in use:

- Election at Louvain University Princeton
- Election of the IACR board (major association in Cryptography)

Designed for low-coercion environments (not receipt-free).
Crypto toolbox: public-key cryptography

Two keys:
- public encryption key
- private decryption key
Crypto toolbox: public-key cryptography

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- public encryption key
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Based on the notion of **one-way function**:

\[ f : \text{easy to compute} \quad f^{-1} : \text{hard to compute} \]
Crypto toolbox: public-key cryptography

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Examples:
- The discrete logarithm problem (ElGamal encryption)
  For a generator \( g \) of a cyclic group: \( f(a) = g^a \)
- Factoring (RSA encryption)
  Given two (large) primes \( p \) and \( q \): \( f(p, q) = p \times q \)
Behavior of Helios (simplified)

Phase 1: voting

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$v_A = 0$ or $1$

$v_B = 0$ or $1$

$v_C = 0$ or $1$
Behavior of Helios (simplified)

Phase 1: voting

```
\{v_D\}_{pk(E)} \rightarrow \\
Bulletin Board
\begin{array}{l|l}
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\text{Chris} & \{v_C\}_{pk(E)} \quad v_C = 0 \text{ or } 1 \\
\end{array}
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\( pk(E) \): election public key
Behavior of Helios (simplified)

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Behavior of Helios (simplified)

Phase 1: voting

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*pk(E): election public key; private key shared among trustees.*
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\(v_A \in \{0, 1\}\)
\(v_B \in \{0, 1\}\)
\(v_C \in \{0, 1\}\)
\(v_D \in \{0, 1\}\)

Phase 2: Tallying using homomorphic encryption (El Gamal)

\[
\prod_{i=1}^{n} \{v_i\}_{pk(E)} = \left\{ \sum_{i=1}^{n} v_i \right\}_{pk(E)}
\]

Based on \(g^a \ast g^b = g^{a+b}\)

\(\rightarrow\) Only the final result needs to be decrypted!

\(pk(E)\): election public key; private key shared among trustees.
This is oversimplified!

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Result: $\{v_A + v_B + v_C + v_D + \cdots\}_{pk(E)}$

In Helios: use Zero Knowledge Proof $\{v_D\}_{pk(E)}$, ZKP {$v_D = 0$ or $1$}
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\( V_A = 0 \) or \( 1 \)

\( V_B = 0 \) or \( 1 \)

\( V_C = 0 \) or \( 1 \)

\( V_D = 100 \)

Result: \( \{ V_A + V_B + V_C + 100 + \cdots \} \_{pk(E)} \)

A malicious voter can cheat!
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**Result:** \[\{v_A + v_B + v_C + v_D + \cdots\}_{pk(E)}\]

*A malicious voter can cheat!*

In Helios: use Zero Knowledge Proof

\[\{v_D\}_{pk(E)}, \text{ZKP}\{v_D = 0 \text{ or } 1\}\]
Crypto toolbox: zero-knowledge proofs

**Goal**: provide the proof of the truth of a statement, without conveying any information except that the statement is indeed true.

**Example:**

Deciding whether 2 graphs are isomorphic is computationally hard (NP-complete).

How can I prove that I know the isomorphism between G and H without revealing any information about the solution?
Crypto toolbox: zero-knowledge proofs

**Goal:** Alice wants to prove to Bob that she knows the isomorphism $f$ (without revealing $f$)

**Example:**

1. Alice generates a random $f_1$ and computes $I = f_1(G)$. From $f$ and $f_1$ compute $f_2$ such that $f_2(H) = I$.

2. Alice sends $I$ to Bob.

3. Bob randomly chooses $i \in \{1, 2\}$ and sends $i$ to Alice.

4. Alice reveals $f_i$ and Bob checks $f_i(G) = H$

\[
G \xrightarrow{f_1} I \xleftarrow{f_2} H
\]

Alice can cheat with probability $\frac{1}{2}$ by “guessing” the value of $i$.

Iterate $n$ times and reduce probability to $\frac{1}{2^n}$
Vote privacy in Helios?

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Vote privacy in Helios?

Replay attack break vote privacy:
Alice must have voted for the winner!
From Helios to Belenios

Helios does not guarantee **Eligibility verifiability**

~> **ballot stuffing** possible by dishonest Bulletin Board

Developed in the LORIA lab at Nancy.
Supports different tally methods: $k$-out-of-$n$, Condorcet, Majority, Judgment, ... 

https://www.belenios.org/
From Helios to Belenios

Helios does not guarantee **Eligibility verifiability**
\[\leadsto \text{ballot stuffing} \] possible by dishonest Bulletin Board

**Belenios**: variant of Helios

- introduce **credential issuer**
- **public** credentials allow to verify eligibility
- **private** credentials necessary to vote (unknown to Bulletin Board)

Developed in the LORIA lab at Nancy.
Supports different tally methods: $k$-out-of-$n$, Condorcet, Majority Judgment, ...

[https://www.belenios.org/](https://www.belenios.org/)
How do I know whether my voting system is secure?
5.1. Contrôle du protocole cryptographique

| 5.1.1 | Critères de contrôle: le protocole doit être conforme à l'objectif de sécurité et aux hypothèses de confiance figurant dans le modèle abstrait décrit au ch. 4. Pour cela, il doit exister une preuve cryptographique et une preuve symbolique. En ce qui concerne les composants cryptographiques fondamentaux, les preuves peuvent être apportées sur la base des hypothèses de sécurité généralement admises (par exemple « random oracle model », « decisional Diffie-Hellman assumption » et « Fiat-Shamir heuristic »). Le protocole doit se fonder si possible sur des protocoles éprouvés. |
Symbolic protocol verification

Est-ce que le système satisfait la propriété ?

\[ \square \forall z. (\text{end}(z) \Rightarrow \text{begin}(z)) \]

Difficultés : attaquant arbitraire contrôlant entièrement le réseau

Techniques : déduction automatique, logique du premier ordre, model-checking, théorie de la concurrence, ...
Symbolic protocol verification

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Conclusion

- Voting through the Internet is a form of remote voting
- **Distribution of credentials** (login/password) is a sensitive procedure (above all if no existing infrastructure)
- Good **privacy** and **verifiability** guarantees if client is trusted
  \[\Rightarrow\text{malware resistance}\] an active research topic
- **Receipt-freeness** / **coercion-resistance** can be achieved but solutions are generally complicated
- **In cryptography we trust?**
  \[\Rightarrow\text{complicated procedures} \text{ – need to trust experts}\]
“It’s not who votes that counts. It’s who counts the votes.”
—apocryphally attributed to Josef Vissarionovich Stalin,
Soviet revolutionary, political leader, party animal and all-around scary character.

Questions?